**Algebra 2 & Trigonometry - Summer School 2018**

Mr. Williams

Email: Philly\_Williams@horacemann.org

Welcome to Algebra 2 & Trigonometry over the summer! I am dedicated to providing a challenging and thoughtful course in an engaging and supportive environment. I’m deeply invested in developing each student’s ability to think independently and creatively about mathematics. Below is an outline of the course content and materials. Please feel free to contact me if you have any questions or concerns at the email above.

Required Materials and State of Mind

* Text: Algebra and Trigonometry, 2nd edition; Stewart, James. Redlin, Lothar, Watson, Saleem; BROOKS/COLE CENGAGE Learning (Provided For You)
* A TI-83 Plus or TI-84 graphing calculator.
* I advise bookmarking the Desmos Graphing Calculator Page on your home computer’s browser or just get the Desmos Graphing Calculator app for your iphone, android, or iPad.
* I also strongly advise you get GeoGebra 5.0 or above. It is free, just google it! We will use it for many reasons. It looks like Geometer’s SketchPad … but it’s SO much more!
* Pencils with erasers on test days.
* Enthusiasm, good humor and a willingness to work hard all year long. Without enthusiasm for math  and a good work ethic, you wont be happy or successful in this class.
* Compliance with HM’s honor code.

Grading Policy

Your three “trimester grades” will be the average of 3 tests and 2 quizzes that count as half a test each. We will have three quizzes, but the lowest quiz grade will be dropped.

For each test score less than 90%, you may write a test analysis. The guidelines for will be handed out to you when I return your first test. You will earn some valuable points back to your test for doing this reflective assignment.

You may have noticed that there is no “homework” grade. You must do homework to learn the material. It is your sole job day to day to make sure you are doing it, correcting it, and getting help when you need it.

I do reserve the right to raise or lower a trimester grade based on participation, attendance, etc. There are no formal extra credit assignments.

I look forward to our summer together and hope you look back on it as a wonderful time both as a student and as a citizen of our summer community here at HM.

Thank you and cheers to the awesome summer ahead!

~Mr. Williams

Course Content

This course aims to expand upon algebraic expressions and operations through the lens of functions, graphs and real-world models. Specifically, throughout the summer we will explore the following topics through our new lens:

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| --- | --- |
| *Topic* | *Textbook Section* |
| Intro to Functions  | P.2, 2.2, 3.2, 3.3 |
| Modeling, calculators | P.1, 2.3, 3.2 |
| Algebraic Expressions | P.2, P.3, P.4, P.5, P.7 |
| Absolute Value Inequalities | 1.7 |
| Lines and Systems | 2.4, 10.3 |
| Piecewise defined functions | 3.2 |
| Transformations | 3.4, 1.7 |
| Quadratics | P.6, 1.1, 1.3, 1.6, 3.5 |
| Composition of Functions  | 3.6 |
| Inverses | 3.7 |
| Exponential Functions | 5.1, 5.4 |
| Logarithmic Functions | 5.2, 5.3, 5.4 |
| Trigonometry | 6.1, 6.2, 6.3, 7.1, 7.2, 7.3 |

Due to the additional time and small size of the class, other non-traditional (translate: topics I know the students will love and will give greater insights to our primary topics) will also included. These will include, but will not be limited to “graphing polynomial functions of degree three or greater,” and “parametrically defined motion in the plane”, and finally “conic sections,” in particular to studying them through the lens of transformations. Hopefully we will have time to talk about solving for sides, angles and area of oblique triangles as well, but that will be a time dependent decision.

The following pages provide a description of on the topics listed above. These details came from our current Algebra II & Trigonometry team summary:

1. **Introduction to Functions**
2. Functions model situations in the real world. Students should know that there are multiple ways of defining a function. For example, a verbal description, a table of values, or a graph could all define a function. It’s not only algebraic formulas.
3. Intervals and coordinates
4. Definition of set, element,  and . Also, union and intersection. We keep this very simple, only the most basic examples. (Included in §P.2)
5. Intervals on the real number line: definitions, open and closed, graphing intervals, writing them with set-builder notation and inequalities, writing them with interval notation. Finding unions and intersections of intervals. (Included in §P.2)
6. The coordinate plane. Students should know the quadrants, and how to write points as coordinate pairs.
7. Relations and functions – more formal definitions (pp. 231 – 2: “Discovery Project”)

A relation is any set of ordered pairs. A function is a set of ordered pairs where the first coordinates are all distinct. This is the same as saying each element of the domain maps to one element of the range. On the coordinate plane, this is the same as passing the vertical line test.

1. Graphing Toolkit Functions (Section 3.2)

Students should be able to graph the “toolkit” functions on p. 226: x^2, x^3, x^4, x^5, sqrt(x), cbrt(x), 4th root(x), 5th root(x), 1/x, 1/x^2, |x|, greatest integer function. They should be able to do this without a calculator, and should know all the domains and ranges of these functions. They should also be able to graph these functions over limited domains. For example, “Graph ,” or “Graph  over the set of integers.”

1. Graphing non-function relations (Section 2.2)

Students should be comfortable with the idea of non-function relations. For example, they should know how to graph , and compare it to . They should also be able to use an equation to tell if it represents a function or not.

1. Symmetry (Section 2.2 and 3.2)

Odd and even functions, odd and even relations. How to use a formula to know if a function has odd/even symmetry, and how to tell from a graph.

1. Increasing/Decreasing, Max/Min (Section 3.3)

We introduce the formal definitions of increasing and decreasing functions. Students should know that most functions both increase and decrease on different intervals, but some are always increasing/decreasing, and they should be able to recognize real world situations where this occurs. We also teach relative and absolute maxima and minima. They should be able to look at a graph, or to graph a given function on their calculators to find the answers. They should know how to find the max/min value on the calculator.

1. **Functions as Models, Graphs, Calculators**
2. Getting information from a graph

Students should be able to look at the graph of a function and determine the following:  for a given *a*, solve  (estimating if necessary), including cases where the solution has multiple values, find the x- and y-intercepts, find max and min values, solve  and similar, find the domain and range. Find the intervals on which the function is increasing and decreasing. On a graph with two functions, solve , , etc. (Section 3.2)

1. Using calculators to solve equations (Section 2.3)

Given an equation or inequality, students should be able to graph the two sides on the calculator and compare them to find the solution.

1. Interpreting function in context

Given a function that represents a real-world situation, students should be able to answer questions posed in the context of the problem, with answers that make sense in context too. For example, given the graph of a function D(t) that represents the distance a person was from home *t* hours after 8 a.m., students should be able to answer questions such as “What was the person doing between 10 and 11 a.m.?” and “at what time were they farthest away from home?” They should also be able to graph a function on their calculator and use the graph there to answer the questions.

1. Find formulas for real-world situations

Given a real-world situation, find the formula for a function that models the situation. Section P.1 has good examples of this, but you need to change the wording to include function notation.

1. **Algebraic Expressions**
2. Review

The following is a list of review topics from Algebra I. We like to mix this in with getting the students accustomed to function notation.

1. Squaring and cubing binomials,
2. Adding polynomials (section P.5)
3. Adding rational expressions (simple problems)
4. Multiplying rational expressions (simple problems)
5. Working with square roots as radical expressions: simplifying square roots, combining radicals
6. Identify rational and irrational numbers (section P.2)
7. Integer Exponents (Section P.3)

Know the meaning of negative exponents, simplify fractions that have positive and negative exponents in the numerator and denominator, work with both numeric and algebraic expressions. Be prepared for a common mistake like .

Include a brief intro to scientific notation.

1. Rational Exponents and Radicals (Section P.4)

Introduce nth roots. Compare the graphs of and  for odd and even values of *n*. Students should know that  from both a numeric and a graphical perspective, and similarly that . Convert between radical form and exponential form, for both numeric and algebraic expressions. Simplify radical expressions involving nth roots, add and subtract radical expressions, simplify numeric expressions with fractional exponents. They do not need to rationalize denominators with nth roots, only with square roots.

1. Rational Expressions (Section P.7)

Simplify rational expressions. Factor quadratics as necessary to help in this process. Add, subtract, multiply rational expressions. Simplify compound fractions, even ones where the “fraction” is given via a negative exponent (i.e., )

This is simply section P.7, up to problem #67 (not the rest of the section).

Also, back to functions – how reducing a factor in a rational function can leave a hole in the graph.

1. Recognizing whether a function is a polynomial function (Section P.5)

Degree, being able to write a polynomial in ascending or descending order

1. **Absolute Value Equations and Inequalities (Section 1.7)**
2. Absolute value as a distance

Interpret absolute value of a number as the distance from the origin on a number line, interpret the absolute value of a difference as a distance between two quantities. Tie this idea to graphs of absolute value functions (perhaps on the calculator).

1. Solve equations and inequalities by interpreting them as a distance

I.e., |x-3| = 5 is the set of points 5 units away from 3, so the solutions are -2 and 8.

1. Solve equations and inequalities in algorithmically

Rewrite the absolute value as two equations/inequalities and solve.

1. Recognize absurd situations

Know when to stop using the algorithm because an answer in impossible anyway.

|x + 4| > -3 is not something you need an algorithm to solve.

1. **Lines and Systems of Linear Equations**
2. Lines (Section 2.4)

Slope of a line, points on a line, x- and y-intercepts, finding the equation of a line given specific conditions, including parallel and perpendicular to a given line, graph a line given in any format. Real-world applications, interpreting slope and y-intercept in context, finding an equation given a value and a rate of change in context.

1. Systems (Section 10.3)

Students should be able to solve a linear system a) algebraically and b) by graphing with the calculator and, when possible, c) by graphing without the calculator. They already know the algebra that they need, so this is somewhat a review. Finding the point of intersection on a calculator will be new to them. In section 10.3, we only do the algebraic problems; the word problems there are not required. However, they do need to be able to solve word problems that involve writing linear functions. For example, compare the costs of two rental cars, with both fixed cost and a cost per mile. Also, they need to be able to find the distance from a point to a line.

1. **Piecewise Defined Functions (Section 3.2)**
2. Graph piecewise-defined functions where the component functions are linear or simple toolkit functions. Also, find the equation of these functions given a graph. Know when to include and exclude endpoints of intervals. Write an absolute value function as a piecewise function.
3. Piecewise Functions in context

Students should understand how piecewise functions appear in the real world, and be able to write and use piecewise functions based on a situation.

1. **Transformations of Functions (Section 3.4)**
2. The Transformations

Students should be able to transform any of the toolkit functions as well as any function given in a graph. They need to be able to translate horizontally and vertically, reflect horizontally and vertically, dilate horizontally and vertically, and take the absolute value of a function. They need to be able to combine these transformations in any way, except we do not require that they mix a horizontal dilation and translation. In other words, they do not need to be able to deal with *f*(*ax* + *b*).

1. Other details

Students should be able to transform non-function relations too, such as . They should also know how transformations affect the domain and range of the toolkit functions, or any function. They also need to be able to express a transformation in words, or come up with a formula given a verbal description of what transformation was performed. They should be able to solve equations graphically involving transformed functions (Section 2.3).

1. Absolute Value Equations and Inequalities (Section 1.7)

Use graphs of the form

  and 

to solve absolute value equations and inequalities. (Similar to a topic listed above, but a different method.)

1. **Quadratic Functions and Equations (Sections 3.5 and 1.3)**
2. Graph quadratic Functions (Section 3.5)

Students should be able to graph functions given in any of the following three forms:

 i) vertex form:  (“standard form” in text)

 ii) polynomial form: (often called “general form”)

 iii) factored form: 

Students should able to determine the equation of a quadratic function

 i) given its vertex and another point on the parabola

 ii) given two x-intercepts and a point not on the x-axis

When asked to graph a quadratic function students must be able to

 i) state its domain and range

 ii) state the equation of its axis of symmetry

 iii) state its zeros

 iv) state its y-intercept

 v) state the minimum or maximum value

 vi) state, with both coordinates, the maximum or minimum point

Students should know that the x-coordinate of the vertex can be found using the formula , and that this makes it easier to graph a function in polynomial form.

1. Students should be familiar with the terms “root,” “zero” and “x-intercept” and know how solving the equation f(x) = 0 is the same as finding the x-intercepts of a graph or the zeros or roots of a function.
2. Students should be able to solve quadratic equations f(x) = 0 (Section 1.3)

 i) by factoring (when possible)

 ii) by using the quadratic formula

 iii) by square rooting both sides when in the form  (Section 1.1)

 iv) by completing the square

v) when y=f(x) has x-intercepts to graph  on a calculator and find its x-intercepts

vi) when the equation has non-real solutions (section 1.4)

1. Quadratic Modeling
2. Modeling with quadratic equations (Section 1.3)

Write a quadratic equation that applies to a real-world situation, solve the equation, interpret the solutions in context.

1. Modeling with Quadratic Optimization – Intro (Section 3.5)

Given a word problem that comes with a quadratic formula, find the max or min value, and interpret the answer in context.

1. Modeling with Quadratic Optimization – More (pp. 288 - 301)

Given a word problem, come up with a function that describes the situation. Find the vertex of the function to solve the problem and interpret the solution in context.

In particular, students should be able to work with area problems (i.e., fixed amount of fencing encloses the maximum rectangular area) and revenue problems (i.e., with a linear demand function, find the price that maximizes the revenue).

1. Quadratic Inequalities (Section 1.6)

Students should see that the following endeavors are equivalent where is a quadratic polynomial:

a) Solve for *x*: 

b) Find all values of *x* for which the graph of is above the x- axis.

Students should be able to solve quadratic inequalities where the quadratic in question is easily factorable.

1. Discriminant (Section 1.3, 3.5)

Given that  is a quadratic polynomial with rational coefficients, students should be able to determine, using the discriminant, whether

a)  has exactly one solution

b) is tangent to the x-axis

c) has exactly two real solutions and whether they are rational or irrational

d) has exactly two x-intercepts and whether they are rational or irrational

e) has two imaginary solutions

f) has no x-intercepts

Students should also be able to use the discriminant to come up with a quadratic that has certain properties. For example: Given , find all values of *k* that allow the equation to have two real solutions. Or, similarly: Given the function  find all values of *k* that allow the function’s graph to have two different *x*-intercepts.

1. **Polynomial and rational functions**
	1. Roots of polynomials, behavior at each root, sketching graphs from a factored polynomial (and vice versa)
	2. Rational roots theorem and dividing polynomials with synthetic division to factor
	3. Dividing factored polynomials, sketching graphs, exploring end behavior and vertical asymptotes
2. **Composition of Functions and Inverses**
3. Composition of Functions (Section 3.6)

How to find work with composition of functions numerically, using formulas, and by reading graphs with two functions. The domain of a composition is often not the domain of either of the original two functions. Word problems that involve composing functions.

1. Inverse Functions (Section 3.7)

One to one functions, knowing when the inverse of a function is another function. Some inverses are non-function relations. Restricting the domain so the inverse is a function. How to calculate the inverse of a function algebraically and how to sketch it graphically. In a word problem, how to interpret the meaning of an inverse function.

1. **Exponential and Logarithmic Functions**
2. Exponential Graphs (Section 5.1)

Graph exponential functions and transform the graphs. Compare graphs with different bases, including bases that are reciprocals of each other.

1. Compound interest and *e.* (Section 5.1)

Calculating compound interest, up to an introduction of *e*. However, *e* is not on the final exam.

1. Intro to Logs (Section 5.2)

A logarithm is the inverse of an exponential function. Evaluate logs without a calculator; convert from an exponential expression to a logarithmic expression and vice versa. Work with logs of any base.

1. Graphing Logs (Section 5.2)

Graph a log function, first as a reflection of an exponential function, and then as its own shape. Transform log graphs, know their domain and range. Compare graphs with different bases.

1. Laws of Logarithms (Section 5.3)

Addition rule, multiplication rule, change of base formula.

1. Logarithmic and Exponential Equations (Section 5.4)

Use logs and exponents to solve equations. Find solutions algebraically and graphically on the calculator. Includes solving word problems that give you exponential functions.

1. **Trigonometry**

There are two chapters on trig that we cover in part. These chapters can be repetitive, with concepts from chapter 6 showing up a different way in chapter 7. I find that covering all of it is actually problematic, and confuses the students as they move from one interpretation to another. Pick the parts that you need, go in any order, and just make sure you cover the main ideas in both chapters, whatever approach you take.

1. Angles (Section 6.1)

Radian measure, angles in standard position, arclength and sector area, circular motion

1. Right Triangle Trig (Section 6.2)

The students are already familiar with sine, cosine and tangent. You review those functions, and introduce secant, cosecant and cotangent. Values for special angles (30, 45, 60), using trig to find side lengths, and solve word problems. Define the trig values in terms of side lengths (e.g. ).

1. Trig Ratios on the Coordinate Plane (Section 6.3)

Introduce new definitions of the six trig functions, e.g. . Use reference angles; work with all three Pythagorean Identities; positive and negative trig values in the four quadrants; find all six trig values for an angle given one value and the quadrant; convert from degree-based problems over the radian-based problems

1. The Unit Circle (Section 7.1)

Definition of unit circle, terminal points, “reference number” as opposed to reference angle, coordinates of points with special angles (pi/6, pi/4, pi/3, pi/2).

1. Trig Functions on the Unit Circle (Section 7.2)

Use the coordinates of a terminal point to find the value of the six trig functions, new definitions of trig values (e.g., ). Find the formula for one trig function in terms of the formula for a different trig function.

1. Graphing Sine and Cosine (Section 7.3)

Periodicity, basic shapes, how the graph describes circular motion. Transformations of the graphs that change amplitude, period, and midline. Label the *x*- and *y*-axes with sensible units. Find the equation of a trig function given its graph.

1. Inverse Trigonometric functions

Graphically and algebraically generating the functions. Evaluating for special angles. Domain and range restrictions for composition with sine, cosine, and tangent